Journal of Engineering Physics and Thermophysics, Vol. 72, No. 2, 1999

INVESTIGATION BASED ON THE USE OF SEVERAL MODELS AS A GENERAL METHOD OF PHYSICAL MODELING

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UDC 53:001.891.57

A refined formulation of a theorem on modeling based on the use of several models is presented. It is shown that investigation based on a single model can be treated as a particular case of the method proposed. Advantages of the method developed over existing modeling based on the use of a single object or phenomenon are considered.

Physical modeling consists in replacing investigation of a certain object or phenomenon by an experimental investigation of its model having the same physical nature. It is used in cases where full-scale tests are problematic. Physical modeling is based on application of the theory of similitude and dimensional analysis [1-7]. The necessary conditions of modeling are geometrical similitude (shape similitude) and physical similitude of the model and nature: at similar instants of time and at similar space points values of variables characterizing phenomena in nature should be proportional to those for the model. The presence of this proportionality makes it possible to recalculate experimental results obtained for the model into values for nature by multiplying each of the determined values by a multiplier that is constant for all similarly dimensioned quantities – a similarity coefficient. Since physical quantities are related by certain relationships stemming from laws and equations of physics, one can choose several of them as basic and express the similarity coefficients for all other derivative quantities in terms of the similarity coefficients for the quantities assumed to be basic. It stems from the existence of these relationships that certain dimensionless combinations should exist for a given physical phenomenon that would characterize this phenomenon and would have one and the same value for the model and nature. These dimensionless combinations of physical quantities are called similarity numbers. Their identity for the model and nature is a necessary condition for physical modeling. However, not in all cases can this be achieved, since identity cannot be satisfied for all similarity numbers in all situations.

In view of the limited possibilities of the above modeling for solving elastic problems of pressure processing of metals, a method for evaluation of deformations of an article was proposed in [8] whose realization implies the use of three models. Later, a theorem on modeling based on the use of several models was formulated and proved [9, 10]. In [10, 11], an example of application of the proposed modeling to evaluation of the temperature field of a forming instrument in hot-molding processes was presented, and in [12], the expediency of application of this type of modeling to investigation of processes of pressure processing of metals was pointed out.

The refined formulation of the theorem on modeling based on the use of several models is as follows:

In physical modeling of an object or a phenomenon based on the use of $2n + 1$ models ($n = 0, 1, 2, \ldots$), the quantity determining the sought property of the object or phenomenon being modeled is defined by the relationship

$$A_{\mathrm{n}} = A_{1\mathrm{m}} \frac{A_{2\mathrm{m}(0)}}{A_{3\mathrm{m}(0)}} \frac{A_{2\mathrm{m}(1)}}{A_{3\mathrm{m}(1)}} \cdots \frac{A_{2\mathrm{m}(n)}}{A_{3\mathrm{m}(n)}} , \tag{1}$$

where quantities characterizing the sought property and entering A_i (i = n, 1m, 2m(n), ..., 2m(n), 3m(n)) as parameters should be chosen with complete or partial coincidence of the ratios

Physicotechnical Institute, National Academy of Sciences of Belarus, Minsk, Belarus. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 72, No. 2, pp. 355-358, March-April, 1999. Original article submitted January 9, 1998; revision submitted February 10, 1998.

1062-0125/99/7202-0337$22.00

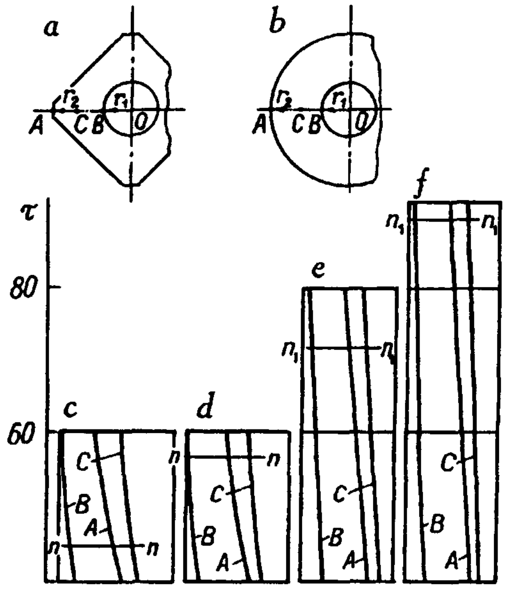

Fig. 1. Top view of experimental samples (a, b) and fragments of oscillograms of temperature variations at points of calking of thermocouples A, B, and C for square-shaped (c, e) and ring-shaped (d, f) samples fabricated from 45-brand steel (c, d) and lead (e, f): $AC = BC$; $n-n$ and n_1-n_1 are lines crossing portions of oscillograms in which the temperatures at points A, B, and C are coincident for the steel and lead samples, respectively. τ, sec.

$$A_n/A_{1m}\,;\;\; A_n = A_{2m(0)}\,;\;\; A_{1m}/A_{3m(0)}\,;\;\; A_{2m(0)}/A_{3m(0)}\,;\;\; \dots\,;\;\; A_{2m(n)}/A_{3m(n)}\,, \tag{2}$$

where the subscripts n *and* m *denote nature and the model, respectively.*

In [9], a proof of the theorem is presented for the case of modeling of elastic translations induced by a gradient temperature field in a flat body with an opening. A proof of the theorem not connected with consideration of a particular object or phenomenon under investigation is given in [10]. It is based on the assumption of the possibility of representation of the quantity under investigation in the form of a product of quantities each of which depends only on a single parameter of the process being considered. The possibility of this representation of the quantity

$$a = f(a_1, a_2, \dots, a_k)\,, \tag{3}$$

which is a function of the mutually independent dimensioned quantities a_i, is substantiated, e.g., in [7], where functional dependence (3) is represented by the product

$$a = c\, a_1^{m_1} a_2^{m_2} \dots a_k^{m_k}\,, \tag{4}$$

where c is a dimensionless constant, and the exponents m_1, m_2, ..., m_k are determined by the formula for the dimension of a.

It should be noted that the validity of the modeling proposed was also tested using experimental data on nonuniform heating of two pairs of samples with a similar shape (see Fig. 1a, b) manufactured from 45-brand steel and lead [13]. To do this, we measured the heating time of the samples required for establishing identical temperature fields in each of the pairs manufactured from steel and lead along chosen directions (see Fig. 1c-f), which were detected using thermocouples and an N04/U4.2 oscilloscope. The models satisfied conditions (2) and were characterized by the following parameters: shape, dimensions, material, and the original thermal state of the samples; the conditions of heating and recording temperature changes and the heating time. The sought heating time of the samples was evaluated from oscillograms of changes in the temperature field in the samples and the feeding rate of the photosensitive paper in the oscilloscope. The values found for the heating time of the samples were inserted in Eq. (1). Here the right- and left-hand sides of relationship (1) coincided with an accuracy not worse than 8%, which corresponded to the experimental errors in evaluation of the heating time of the samples.

In the case of complete coincidence of the parameters in relationships (2), Eq. (1) simplifies to the form

$$A_{\mathrm{n}} = A_{1\mathrm{m}} \tag{5}$$

This allows passage to an investigation on a model mimicking nature.

In the case of partial coincidence of the parameters in relationships (2), quantities A_i evaluated experimentally (or using mathematical models) allow calculation of the following ratios:

$$A_{2\mathrm{m}(0)}/A_{3\mathrm{m}(0)}\,;\; A_{2\mathrm{m}(1)}/A_{3\mathrm{m}(1)}\,;\; \dots\,;\; A_{2\mathrm{m}(n)}/A_{3\mathrm{m}(n)}\,, \tag{6}$$

which in fact are similarity numbers for nature and the model in determination of the quantity A_{n} from $A_{1\mathrm{m}}$. When these numbers are known, modeling based on the use of several models is reduced to modeling based on the use of a single model. It should be noted that the values of these ratios for each spatio-temporal coordinate of the 1m-model can differ from one another.

The absence of the necessity of choosing a single definite model satisfying the similarity numbers and the lack of knowledge of these parameters, which should be either determined experimentally or calculated based on already known laws, are important distinctions of modeling based on the use of several models compared to modeling based on a single model. In addition, as has been stated above, in modeling based on the use of a single model, not all similarity numbers can be satisfied in all cases, which in realization of such an experiment will lead to errors that virtually cannot be estimated, or even to rejection of the modeling.

Obtaining incorrect results in modeling of objects or phenomena based on the use of a single model upon changing its size with respect to nature is well known. This leads to the necessity of full-scale tests, as, e.g., in the aircraft industry, and a substantial increase in research expenses. Increasing the scale of photographs of, e.g., shots of moiré bands using an enlarger, in order to facilitate their processing, can lead to conclusions that are far from the actual picture (the author encountered this phenomenon in processing moiré patterns with the aim of elucidating the effect of reinforcement on the size of molding dies). Most likely, upon a change in the scale of the spatio-temporal coordinates, in certain cases the phenomenon or object is characterized by parameters that can substantially affect the sought charactersitic A_{n}. This aggravates the problem of correct evaluation of similarity numbers and specification of their values in existing modeling.

However, the proposed method of modeling removes possible errors in its implementation even upon changes in the scale of several models, since, according to the theorem on modeling based on the use of several models, each of the parameters of nature should be included without changes in at least one of the models used in the modeling. Here it allows the use of models convenient for investigation, which can also be regarded as a strong feature of the method compared to existing modeling.

The author supposes that in the modeling proposed, similarity is applied not only to similar instants of time and space points of individual parameters of the object (or phenomenon), as is done in the case of existing modeling, but also to the object (or phenomenon) itself presented in new spatio-temporal coordinates, which provides for correct application of the theory of similitude.

Thus, modeling based on the use of several models is a general method of physical modeling. The use of all parameters characterizing nature in models chosen in accordance with the theorem provides the possibility of rejecting a single model given by similarity numbers in an existing method of modeling and obtaining reliable results.

NOTATION

A_i, quantities of the sought property of the object or phenomenon; i = n, 1m, 2m(0), ..., 2m(n), 3m(n); n = 0, 1, 2, ...; n and m, subscripts denoting that the object belongs to the natural sample and the model, respectively;

a, quantity being investigated; a_k, dimensioned quantity; c, dimensionless constant; m_k, exponent; τ, heating time of the sample to a certain temperature.

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